4.7 Identity and Inverse Matrices

Identity Matrix--square matrix that when multiplied by another matrix, it equals that same matrix.

$A \cdot I = A \qquad I \cdot A = A$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse matrices--are 2 square matrices whose product is the identity

A^{-1} -- "A inverse"

$$A \cdot A^{-1} = I$$

$$A^{-1} \cdot A = I$$

Are they inverses? yes

$$A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Are they inverses? No

$$C = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$$

$$C \cdot D = \begin{bmatrix} 1 & -13 \\ & \end{bmatrix}$$

Finding the inverse.

If D = 0, there is no inverse.

A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $A^{-1} = \frac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

ad-bc

Find the inverse.

$\begin{bmatrix} 6 & 4 \\ -1 & 3 \end{bmatrix}$ $\frac{1}{22}\begin{bmatrix} 3 & -4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} \frac{3}{22} & -\frac{2}{11} \\ \frac{1}{22} & \frac{3}{11} \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 8 & -2 \end{bmatrix}$ $\frac{1}{2}\begin{bmatrix} -2 & 0 \\ -8 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -4 & -\frac{1}{2} \end{bmatrix}$

$\begin{bmatrix} -4 & 6 \\ -2 & 3 \end{bmatrix}$ No Inverse

Cryptography

_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

13|38|24|49|44|107|19|57|22|53|17|39

A= $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ $A \cdot B^{-1}$

To decode A^{-1}

$M \times A^{-1}$

6x2 2x2

A B

$\begin{bmatrix} 13 & 38 \\ 24 & 49 \\ 44 & 107 \\ 19 & 57 \\ 22 & 53 \\ 17 & 39 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}^{-1}$

ALWAYS_SMILE

_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

EDDY

5|4|4|25

$\begin{bmatrix} 5 & 4 \\ 4 & 25 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 22 \\ 29 & 83 \end{bmatrix}$

9|22|29|83

$\begin{bmatrix} 9 & 22 \\ 29 & 83 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}^{-1} =$

_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

14|29|28|64|20|60|24|67|20|40|29|72|20|60|26|
57|23|50|26|77

HW
p199
10-13, 16-25